Mathematical Model of Economic Forecasting and Analysis of the Market Competitive Mechanism

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Abstract

The mathematical models of forecasting depend upon the managers conception regarding the actions development for long – term forecasting or for short – term forecasting.

To this end, we distinguish two directions: an explorative one and a normative one.

From the point of view of the investigating way, the methods and models of forecasting can be synthetic (aggregated) and morphologic, with a high disaggregating degree.

If we take into consideration the nature of information that can be exact, incomplete or uncertain, it results that also the worked out models can be: deterministic, fuzzy, respectively, probabilistic ones (hybrid models can exist too).

The deterministic forecasting is restricted to a single variant.

The study presented in the paper is based on a deterministic hybrid model.

1. Introduction

The system analysts, with a view to working out the mathematical model of the market competitive mechanism, are deeply interested in the physical components of the analyzed economic system.

The linguistic description defines the initial state of the system components by means of the state variables and parameters determined at moment t_0.

Subsequently, investigations are made on the evolution tendencies at moments t_0 for short – term forecasting and for long – term forecasting.

The right estimation of state parameters and variables is followed by the transformation of the simple linguistic description into coherent system of mathematical relations.

The redundancy / non-redundancy, compatibility / incompatibility of the system of relations are verified.

In case of a system of relations without solutions, the most restrictive levels are relaxed by supplementing the resources.

The penalty cost of the supplementary resources is usually proportional to the dual variable attached to the relaxed constraint.

In case of the market mechanism, the system of relations studying the equilibrium between consumption and production is defined by linear variations and the variables of the system are exactly the probabilities τ_{ij}.

The existence of a certain "crises " related to the disciplines using the mathematical modeling was signaled by many specialists in 1980 year, during the IV-th European Congress of operations research, developed in England.

The used optimization methods, based on normative methods, searching the optimum solution, became rigid, far away from the economic reality, practically, not responding to the requirements and obliged the analyst to pay more attention to the art of modeling.

With a view to knowing the laws which define a certain studied economic phenomenon and their utilization to satisfying the proposed objectives, the following stages are considered:
1. Observing the phenomena under the descriptive – qualitative aspect (causality among phenomena);
2. Stating some descriptive – qualitative type laws;
3. Observing the phenomena under quantitative aspect;

The variables are quantitatively defined by two means, namely: a statistic study made for a past significant period and the organization of certain experiments to working out a system or working hypotheses are stated.

4. Stating some quantitative laws. The obtained data are analyzed and, on their basis, the first hypotheses on the laws which could govern the analyzed system, are stated.
5. Adopting certain action decisions on nature, with a view to directly or indirectly satisfying the human needs;
6. Monitoring the effects of the decisions made and improving the way of future decision-making.

All these aspects can be synthesized and used into an economic – mathematical model. The model construction depends upon the collected information, especially the numerical ones (initializing data, coefficients or parameters intervening in the functions describing a certain law or constraints).

Several algorithms are used to solving the model.

The procedural modeling is characterized by firstly emphasizing the algorithm and secondly, the model.

The procedural modeling can be fulfilled in two strategies, namely:

** general modeling when we have in view to catch all possible cases;*

** modeling on types of problems (classes), when we choose problems frequently met in practice, for which a specific solving algorithm is worked out.*

Since in economy, there is a tight connection among the used methods and the nature of the values characterizing the analyzed process, the more exactly the values can be measured the more rigorously will be the methods used to decision-making. Under these conditions, **exact (rigorous) algorithms** will be used.

On the other side, if we have exact data, but the problem is complex, of high dimensions or the input data are inexact, (of stochastic or fuzzy nature) we apply to heuristic algorithms.

In fact, the heuristic methods are applied by people to their activities, mostly, they not being aware of this thing.

The problem is to discover the basic rules used in the specialist's heuristics, to improve them and to systematize their application using an algorithm.

Most authors (Simon H., Heinz Klein etc.) agree that heuristics represents an ensemble of methods which allow to obtaining certain "**good"** solutions. We mean by this, the fact that the solutions, even if they do not always take rigorously into consideration all the required conditions, and the respective deviations are difficult to be estimated, yet, from a practical point of view, according to the efficiency criterion and after a minimum number of requirements considered as priority, they can be accepted as good enough.

We mean by a "**good**" rule, that rule satisfying from two points of view:

** the quality of the obtained solution (namely, a feasible deviation as against the true solution);*

** the use of certain calculation resources according to available bounds (as calculation time, storage etc.).*

It results that the fuzzy elements of heuristics are, in fact, amplified by the above statements. Additionally, different parameters which can be only conventionally taken into consideration, will intervene to getting "good" solutions .

Consequently, according to this signification of heuristics, numerous fuzzy elements are included, beginning with the definition and finishing with the description of the used rules. The rules used to solving a problem can be:

* elementary (independent) rules;

* compound rules (dependent upon the elementary rules).

By neglecting a rule *f*, we obtain an antithetic rule $\overline{f}$. For instance, in scheduling, by neglecting the SPT rule (shortest processing time) , we obtain the LPT rule (longest processing time). A compound rule can be constructed by compounding the rules *f* and $\overline{f}$, using a deterministic operator, for instance, a mixture under the form:

$$\alpha f + (1-\alpha)\overline{f} \text{, where } 0 \le \alpha \le 1.$$

But, generally, even the compounding operator can have a fuzzy character.

This can be achieved by the fuzzyfication of a parameter intervening in the definition of the compounding operator (in the given example, α will be a fuzzy variable).

Another example can be given to compounding the FIFO rule (first come, first served) with the SPT rule. The compounding fuzzy operator is under the form:

** if the queuing length is not too "long", FIFO rule is applied;*

** if the queuing length becomes too "long", SPT rule is applied;*

The "great" fuzzy variable depends on context – in certain cases it means "around 10 persons" , in other cases, "around 15 persons" etc.

The compound rules are under the form:

$F_j(f_{i1}, f_{i2}, ..., f_{ip})$, where F_j is a compounding operator (that can be an analytical function or a very complicated procedure) and $f_{i1}, f_{i2}, ..., f_{ip}$ a subset of the elementary rules set.

It follows that the number of compound rules can be infinite. A membership degree μ can be attached to each independent rule f_i , related to the property to be applied. Similarly, a membership degree $x_{ji1/2}....i_p$ can be attached to a compound rule $F_j(f_{i1}, f_{i2}, ..., f_{ip})$ related to the property to be applied.

The following definition can be given for the open heuristics:

We call an open heuristics a fuzzy set $\ddot{E}$ of elementary (independent) and compound rules , under the form:

$$\overline{E} = \begin{Bmatrix} f_1, f_2, ..., f_n, ..., F_j(f_{i_1}, f_{i_2}, ..., f_{i_p})... \\ \mu_1, \mu_2, ..., \mu_n, ..., \quad x_{j_{i_{1/2}}}, ..., x_{i_p} \; ... \end{Bmatrix} \tag{1}$$

Hence, it follows:

$$f_i \underset{\mu_i}{\in} \overline{E} \tag{2}$$

namely, " the rule f_i belongs to an extend equal to μ_i to heuristics $\ddot{E}$ " or " the rule f_i is applied to an extend μ_i within heuristics $\ddot{E}$ ".

At the same time, we can also say that "Heuristics $\ddot{E}$ is an open set of "good" rules to solve a certain problem". This "set" is defined by using an adjective "good" that introduces a very high degree of imprecision.

The evaluation model of the market shares of the economic agents is constructed by using the Markov chains theory.

Let "n" be the producers and $p_1, p_2, ..., p_n$ and "*m*" the consumers , with centers $C_1, C_2, ..., C_m$.

The production, respectively, the consumption refers to the same product (or group of products of the same functions). The products can be: building materials, prefabricated pieces, buildings, roads etc.

Let N_{ij} be the flow of products manufactured by manufacturer P_j delivered to consumer C_j. Let CT_i be the total solvable demand of the consumer C_i. Let Q_j be the total amount of deliveries of producer P_j. Let Q be the total quantity of products effectively delivered by producers, that is, by hypothesis, equal to the total solvable demand (consequently it is equilibrated).

Thus, a table under the following form is obtained:

Table 1: Consumptions of materials

	p_1	p_1	$\cdots$	p_n	Solvable Total Consumption
C_1	N_{11}	N_{12}	$\cdots$	N_{1n}	CT_1
C_2	N_{21}	N_{22}	$\cdots$	N_{2n}	CT_2
.	.	.	.	.	.
C_m	N_{m1}	N_{m2}	$\cdots$	N_{mn}	CT_m
Total	Q_1	Q_2	$\cdots$	Q_n	Q

The measuring units in this table can be: physical, conventional or value.

The equilibrium relations in this table are:

$$
\begin{aligned}
&N_{11} + N_{12} + \ldots + N_{1n} = CT_1\\
&N_{21} + N_{22} + \ldots + N_{2n} = CT_2\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&N_{m1} + N_{m2} + \ldots + N_{mn} = CT_m \qquad (3)\\
&N_{11} + N_{21} + \ldots + N_{m1} = Q_1\\
&N_{12} + N_{22} + \ldots + N_{m2} = Q_2\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&N_{1n} + N_{2n} + \ldots + N_{mn} = Q_n
\end{aligned}
$$

In the conditions of equilibrating the total supply with the global consumption, we yield:

$$Q_1 + Q_2 + \ldots + Q_n = CT_1 + CT_2 + \ldots + CT_m = Q \qquad (4)$$

Denoting by p_j the market share of producer j, we have:

$$p_j = \frac{Q_j}{Q} \qquad (5)$$

If we denote by a_{ij} the probability as a consumer C_i to purchase products from supplier p_j, we get:

$$a_{ij} = \frac{N_{ij}}{CT_i} \qquad (6)$$

where: $\sum_{j=1}^{n} a_{ij} = 1, (\forall) i \in (1, 2, ..., m)$ (7)

The weight of the demand q_i of the consumer *i*, in the total solvable demand Q is yielded by relation:

$$q_i = \frac{CT_i}{Q} \qquad (8)$$

We mention that from the point of view of the consumer, we have to take into consideration the potential demand, that represents the real level of his needs, regardless of the supply and/or solvability. The total potential demand of consumer i , (CP_i) covers accordingly, the solvable demand (CT_i) and the unsatisfied demand (CN_i) , namely:

$$CP_i = CT_i + CN_i \qquad (9)$$

Similarly, the total capacity of producer *j* (QP_j) includes the real supply (OF_j) and the unused capacity (QN_j), namely:

$$QP_j = OF_j + QN_j \qquad (10)$$

The real supply includes the delivered production Q_j and the stock S_j of undelivered products (at the end of the year), namely:

$$OF_j = Q_j + S_j \qquad (11)$$

In a first phase of modeling, the effect of the existence of the unsatisfied demand (CN_i) and of the undelivered products stock S_j was neglected.

The market share p_j of firm *j*, can be taken as a probability. Really, p_j is the probability as a consumer to purchase products from firm *j*. It is easy to show, according to relations (4) and (5), that:

$$p_1 + p_2 + \ldots + p_n = 1 \qquad (12)$$

It results from (12) and (5) that:

$$p_j = \frac{N_{1j} + N_{2j} + \ldots + N_{mj}}{Q} \qquad (13)$$

It follows from (5) that:

$$N_{ij} = a_{ij}C_{ti} \qquad (14)$$

Replacing in (13) and taking into account (14), we get:

$$p_j = \frac{N_{1j} + N_{2j} + ... + N_{mj}}{Q} \quad (15)$$

It follows from (6) that:

$$N_{ij} = a_{ij} C_{ti} \quad (16)$$

Replacing in (15) and taking into account (14), we obtain:

$$P_j = \frac{a_{1j}CT_1 + a_{2j}CT_2 + ... + a_{mj}CT_m}{Q} = \quad (17)$$
$$= a_{1j}q_1 + a_{2j}q_2 + ... + a_{mj}q_m$$

We denote by p and q the vectors:

$$p = \begin{pmatrix} p_1 \\ p_2 \\ . \\ . \\ . \\ p_n \end{pmatrix} \qquad q = \begin{pmatrix} q_1 \\ q_2 \\ . \\ . \\ . \\ q_m \end{pmatrix} \quad (18)$$

Let A be the matrix of probabilities as firm j to deliver products to consumer i, namely:

$$A = \left\| a_{ij} \right\|, i = 1, 2, ..., m;\ j = 1, 2, ..., n \quad (19)$$

Let A^* be the transposed form of this matrix. Using these notations, relation (17) becomes:

$$p=A^*q \quad (20)$$

It follows from relations (6), (7),(8) that:

$$q_i = \frac{CT_i}{Q} = \frac{N_{i1} + N_{i2} + ... + N_{in}}{Q} =$$
$$= \frac{N_{i1}}{Q} + \frac{N_{i2}}{Q} + ... + \frac{N_{in}}{Q} =$$
$$= \frac{N_{i1}}{Ct_i} x \frac{CT_i}{Q} + \frac{N_{i2}}{CT_i} x \frac{CT_i}{Q} +$$
$$... + \frac{N_{in}}{CT_i} x \frac{CT_i}{Q} = \quad (21)$$
$$= \frac{N_{i1}}{CT_i} x \frac{CT_i}{Q_i} x \frac{Q_1}{Q} + \frac{N_{i2}}{CT_i} x \frac{CT_i}{Q_2} x \frac{Q_2}{Q} +$$
$$... + \frac{N_{in}}{CT_i} x \frac{CT_i}{Q_n} x \frac{Q_n}{Q} =$$
$$= a_{i1}K_{i1}p_1 + a_{i2}K_{i2}p_2 + ... + a_{in}K_{in}p_n$$

where we denoted by:

$$K_{ij} = \frac{CT_i}{Q_j} \quad (22)$$

representing the ratio between the consumer's demand and the total supply of producer j.

Denoting by:

$$\bar{a}_{ij} = a_{ij}K_{ij} \quad (23)$$

we obtain the matrix $\bar{A}$, of components:

$$\bar{A} = \left\| \bar{a}_{ij} \right\| \quad (24)$$

Relation (21) becomes:

$$q_j = \bar{a}_{i1}p_1 + \bar{a}_{i2}p_2 + ... + \bar{a}_{in}p_n \quad (25)$$

that is written under a matrix notation:

$$q= \bar{A}\,p \quad (26)$$

We get from (20), (26):

$$p=A^*\,q=A^*\,\bar{A}\,p = \tau\, p_i \quad (27)$$

where we denoted by:

$$\tau = A^*\,\bar{A} \quad (28)$$

The matrix τ has the components τ_{ij}, representing the probability as a consumer who was a client of producer i to become a client of producer j (called transition probabilities). The system of equations (27) is written under the form:

$$\begin{aligned} &(1-\tau_{11})p_1 - \tau_{12}p_{12} - ... - \tau_{i2}p_{i2} - ... - \tau_{in}p_n = 0 \\ &-\tau_{21}p_1 + (1-\tau_{22})p_{22} - ... - \tau_{2n}p_n = 0 \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &-\tau_{n1}p_1 - \tau_{n2}p_2 - ... + (1-\tau_{-})p_n = 0 \end{aligned} \quad (29)$$

An homogeneous system of "n" equations with "n" unknowns whose determinant is null, was obtained. It results that the system admits an uncommon solution.

By attaching equation (12) and eliminating the last equation as being redundant, the system (30) will have an unique solution:

$$\begin{aligned} &(1-\tau_{11})p_1 - \tau_{12}p_{12} - ... - \tau_{in}p_n = 0 \\ &-\tau_{21}p_1 + (1-\tau_{22})p_{22} - ... - \tau_{2n}p_n = 0 \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &-\tau_{n-1,1}p_1 - \tau_{n-1,2}p_2 - ... + (1-\tau_{n-1,1}) - \tau_{n-1,n} = 0 \\ &p_1 + p_2 + \ldots + p_n = 1 \end{aligned} \quad (30)$$

If we know the transition probabilities τ_{ij}, the market shares p_j of producers are obtained.

In case when the transition probabilities τ_{ij} are constants in time, the competitive mechanism is a stationary process.

We meet this situation only after the fulfilling certain ideal conditions as: balancing the demand with supply, acquiring modern technologies by all producers, identifying the consumers' preferences etc. Additionally, even if this situation exists at a certain moment, it can not last long time, due to technical progress, changes in consumer's preferences and finally, the appearance of the disequilibrium between demand and supply.

Consequently, the matrix τ_{ij} is a time function, so, the competitive mechanism is a nonstationary process. In order to outline the time dependence, we shall attach index t to all parameters which describe the competitive mechanism. The attraction probability a_{ij}^{o} can be calculated at the initial moment $(t = 0)$.

Expanding the presented theory, we can study the evolution of a stationary process of the competition among several producers and we can determine the clients' attraction by using a Cobb – Douglas multiplicative production function.

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