(1+1)-EA for Econometric Models. The Comassability of the Matrix of States

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Abstract

Many topics related to stability of equilibrium, equilibrium selection, transitional dynamics, the long-run evolutionary dynamics of economic processes or other microeconomic or macroeconomic concepts are of the most importance in economic studies and often involve a set of optimization techniques able to provide the best solution under specific conditions.

One of the main issues that should be studied as an optimization algorithm is applied is the total time required for that process to get the optimum. Even in this era of the New Technologies, as computers are designed to work faster and faster, it is welcomed to know to which extent a specific function is welcomed to be optimized by using a specific algorithm, or to which extent the time required by that algorithm to get the optimum is a polynomial or an exponential one.

Among the classical methods that we can use to drastically reduce the dimension of the transition matrix P of the Markov chain attached to (1+1)-EA, the comassability of the states is a very important one. For some fitness functions this comassability is possible, but for other it is not. This paper aims at making an exact description of certain unimodal functions which lead to a transition matrix P comassable in relation with a partition of the states space.

Keywords: Evolutionary Algorithm, time to get the optimum, commasability

1. Introduction

Many topics related to stability of equilibrium, equilibrium selection, transitional dynamics, the long-run evolutionary dynamics of economic processes or other microeconomic or macroeconomic concepts are of the most importance in economic studies and often involve a set of optimization techniques able to provide the best solution under specific conditions. Evolutionary Algorithms can be applied to get solutions for this sort of economic problems and it is proved that in many situations they can lead to good results (see for example [1]).

In fact, it is widely accepted that Evolutionary Algorithms are very important tools in Econometrics, as some models have been proven to be computationally intractable sometimes, due to their complexity (see [5]). Under such conditions, some Evolutionary Algorithms have been taken into consideration by specialists as method for optimization of various econometric models (see for example [3] or [11]) and (1+1) EA is one of them. In spite of the fact that this algorithm is a very simple one, some authors agree with the possibility of using simple versions of Evolutionary Algorithms to describe more complicated forms of them and to solve difficult problems using this sort of methods (see [7]).

Meanwhile, one of the main issues that should be studied as an optimization algorithm is implemented is the total time required for that process of minimizing or maximizing to get the optimum. Even in this era of the New Technologies, as computers are designed to work faster and faster, it is more than welcomed to know to which extent a specific function is recommended to be optimized by using a specific algorithm, or to which extent the time required by that algorithm to get the optimum is a polynomial or an exponential one (see [6], [9] or [10]). In this context, we will focus our attention on one of the classical methods that we can use to drastically reduce the dimension of the transition matrix P of the Markov chain attached to (1+1)-EA, is the comassability of the states. For some fitness functions this comassability is possible, but for others it is not. This paper aims at making an exact description of certain unimodal functions which lead to a transition matrix P comassable in relation with a partition of the states space.

2. Preliminary results

We already know that a transition matrix P is comassable in relation with a partition S = S(1)US(2)U…US(m) of the states space, if and only if the probability value p(x, S(j)) is the same, for every x from S(k) and for every pair of sets S(k) and S(j) where k and j rank between 1 and m. We mention that the results from theorems 2.1 and 2.2 are classical results used in EA study (see for example [9]) and their presentation is due to the fact that the proofs are personal and use some techniques which are very important for the future research papers. We will use the notation B(L) as the product {0, 1}x{0, 1}x … x {0, 1} for L times.

Theorem 2.1. The Mutation Matrix M is comassable in relation with the partition S as above, where we define the S(i) as the set of all x from B(L) with unitary norm and the mutation operator m(x, y)

defined as 1-q if x = y, q/L if the Hamming distance between x and y is equal to 1 and 0, otherwise.

Here, q ranking between 0 and 1 it is a probability value, and x and y belong to the defined space.

Proof:

Let $S(i)$ and $S(j)$ two sets of the partition S and x from $S(i)$ be arbitrarily fixed. Then:

M comassable iff $m(x, S(j))$ has the same value, for any sets of the partition S and for every x from $S(j)$.

We have:

$m(x, S(j)) = \sum m(x, y)$ where y takes all possible values in $S(j)$ and the distance between x and y is equal to 1.

For x belonging to $S(i)$ fixed, we want to find the number of the y from $S(j)$ so that the distance between x and y is 1. But, if this distance is 1, we see that x and y must have only one different position, so y belongs to $S(i-1)$ or to $S(i+1)$. Then,

$m(x, y) = 0$, if $j \neq i-1$ and $j \neq i+1$

We compute the number of y belonging to $S(i-1)$ and to $S(i+1)$ so that the distance between x and y is equal to 1. Every x from $S(i+1)$ has exactly $i+1$ positions with 1, so that the number of y from $B(L)$ with distance between x and y is 1, is $L-i$. Then, $m(x, S(i+1)) = q(L-i)/L$, value which does not depend on the particular choice of x from $S(i)$. The same argumentations show that $m(x, S(i-1)) = qi/L$ and this value is also independent from x belonging to $S(i)$.

Then: $m(x, S(j))$ takes the following values: 1-q, if j=1; qi/L if j=i-1; q(L-i)/L, if j=i+1 and, of course, 0 otherwise.

The values obtained above only depend on the index i and not on the particular individual x, then the matrix M is comassable in relation with the partition mentioned.

Lemma 2.1. For every $S(i)$ defined as above, the distance between two arbitrary x and y from $S(i)$ takes an odd value.

Proof:

If x=y, then the distance is 0, so it takes an odd value .

If $x \neq y$, then we can state that there exists a value k belonging to the set $\{1, 2, \ldots, L\}$ so that the k positions of x and y are distinct, hence the absolute value of the difference between these values is 1. It is obvious that, starting with an x from $S(i)$ arbitrarily fixed and modifying its positions "1", we'll generate the whole set $S(i)$, hence we can obtain y too.

Generally, changing k positions of 1 with k positions of 0 leads to an individual y so that the Hamming distance between x and y being 2k, if 2k≤L. q.e.d.

We define now the set $Di(x)$ as the set of all y from $B(L)$ so that the distance between x and y is 1.

Proposition 2.1. Let x from $B(L)$ be arbitrarily fixed. Then, $Di(x)=x+S(i)$, for every i belonging to $\{0, 1, 2, \ldots, L\}$, where $x+S(i)=\{x+y \mid y$ belongs to $S(i)\}$.

Proof:

Let x from $B(L)$ and z from $S(i)$. Then, there exists two subsets, A and B, of $\{0, 1, 2, \ldots, L\}$, say: $A=\{j_1, j_2, \ldots, j_k\}$, $B=\{r_1, r_2, \ldots, r_i\}$, so that: $x=\sum e(s)$, s belongs to A and $y=\sum e(t)$, t belongs to B. Here, $e(s)$ is the s – row in the unit matrix of L dimension.

The equality of the two sets results, by considering four cases: $A \cap B$ is empty, A is included into B, B is included into A, and the case four, where the previous three situations are not true. All the studies cases lead to the same conclusion, that $Di(x)=x+S(i)$, for every i belonging to $\{0, 1, 2, \ldots, L\}$. Q.e.d.

Generalize to Lemma 2.1. Let x' from $B(L)$ be an arbitrary element of the space. Then, for any x and y from $Di(x')$, the distance between x and y takes an odd value.

Theorem 2.2. The Mutation Matrix $M=(m(x,y))$ is comassable in relation with the partition S and the mutation operator $m(x,y)$ defines as

$$p^{H(x,y)} \cdot (1-p)^{L-H(x, y)},$$

for x, y belonging to $B(L)$. Here, $H(x, y)$ is the Hamming distance between x and y.

Proof:

Let $S(i)$, $S(j)$ two sets of the partition and let x from $S(i)$ arbitrary fixed. Then:

$M(x, S(j)) = \sum m(x,y)$ (2.1.)

Here, y taked all the values in $S(j)$. Let x from $B(L)$, be arbitrary. Then, the number of the individuals which can be at distance Hamming d by x is C_L^d, d from $\{0, 1, 2, \ldots, L\}$ set.

We define, for every d as above, the set Dd(x) with elements y from B(L) such that the distance between x and y takes the value d. (2.2.)

In fact, we defined a B(L) - partition, namely:

D(x)={Dd(x) │ d=0, 1, …, L}   (2.3.)

It can be shown that m(x, S(j)) can be written as

$$\sum card(S(j) \cap Dd(x))p^d (1-p)^{L-d} \quad 2.4.)$$

The problem is, now, to determine the number of the elements of $S_j \cap D_d(x)$, to see that this value is independent from the x choice in S(i). For this purpose, a number of three different cases is taken into consideration, namely j=i, j>i and j<i. This study leads to the conclusion that the conclusion of the theorem is true.

Consequence 2.1. With any mutation operator we'll construct the matrix M, the propriety of comassability is valid in relation with any partition D(x), x from B(L).

Proof:

It is a consequence of the theorems 2.1. and 2.2. and of the proposition 2.1.

3. The comassability of the transition matrix P

In our study we'll start with a classical problem, named the count of the number of 1 from a binary vector. For a (1+1)-EA, this problem asks to minimize the number of zeros.

Let $f : B(L) \rightarrow R$, $f(x) = L - \|x\|_1$. We can see that:

$f|_{S(i)}$ = L-i, for any i from {1, 2, …, L-1}   (3.1.)

We can also see that for any i< j in {1, 2, …, L}, we have

$f|_{S(i)} < f|_{S(j)}$. (3.2.)

In the construction of the matrix P, the relation (3.1.) is very important. The form of the elements of P, we'll see that the chain will never leave a current state x belonging to S(i), unless for transit in a state form a S(k) with k>1.

We can say that p(x, y) is equal to m(x, y) if f(y) < f(x), the same p(x, y) is 0 if f(y) ≥ f(x) and x ≠y, and finally p(x, y) is equal to ∑m(x, z) if y=x and if f(z) ≥ f(x), for any x, y from B(L).

Consequently, the development of the algorithm does not depend on the concrete state x in which we are at a certain point, but on its belonging to a set S_i of the partition of the states of the space.

Băck (1992) found out the elements of the matrix P, and the average numbers of steps necessary to reach the optimum was approximated by Mühlenbein (1992). Both results were obtained by using classical working instruments, but this led to pretty complicated results, so that even for this simple function the theoretical study of (1+1)-EA proved to be difficult.

The key element in the (1+1)-EA description needed for optimizing this function is (3.1). Thus it seems naturally to wonder which the modular functions meeting this condition are and how many such functions exist. Some of the conclusions that derive from studying these functions can be subsequently applied in order to describe similar behaviours for other functions, being them unimodal or not.

Let $f : B(L) \rightarrow R$, be modular.

Proposition 3.1.

$f|_{S(i)}$ = ct, for any i from {1, 2, …, L-1}, then f(x) = $c_0 + c\|x\|_1$ where c_0 and c belong to R

Proof:

"←"is obvious:

$f|_{S(i)} = c_0 + c\|x\|_1$ for any i from {1, 2, …, L-1}, so it is constant on S(i) ,for any i from {1, 2, …, L-1}.

"→" From f modular it follows f linear, so

$f(x)=c_0+c_1x_1+ c_2x_2+…+ c_Lx_L$.

From $f|_{S(i)}$ = ct, for any i from {1, 2, …, L-1} we get

f(e(i))= f(e(j)), for any i≠j, I, j from {1, 2, …, L}

Then: f(e(i))= f(e(j)) ↔ $c_0+c_i= c_0+c_j$ ↔ $c_i= c_j$ for any i≠j . Let c be the common value of these coefficients:

$f(x)=c_0+c(x_1+ x_2+…+ x_{L} = c_0+c\|x\|_1$, q.e.d.

On the set **F** of the functions $f : B(L) \rightarrow R$, we define a relation "≈", thus:

f ≈ g ↔ for any x, y from B(L),   [f(x)<f(y) iff g(x)<g(y)]

It can be proved that this is a relation of equivalence on **F**. In relation with it, the set **F** can be divided into

classes of equivalence. The next problem which is naturally raised refers to the structure of the class measured by a modular function like the one in the above proposition. It can be proved that any element from a class generated by a modular is a unimodal function. But, if we proved that all the elements that belong to f for $f(x)=c_0+c\|x\|_1$ where c_0 and c belong to R are modular functions, we would draw the conclusion that we have not found any extra information about the number of unimodal functions for which the application of the algorithm is justified. The next example will prove that:

Proposition 3.2. If $f : B(L)\rightarrow R$ having the form $f(x)=c_0+c\|x\|_1$, c_0 and c belong to R, then there exists g from the class f^ of f, with g unimodal but no modular**.**

Proof:

Let $f : B(L)\rightarrow R$, $f(x)=c_0+c\|x\|_1$, c_0 and c belong to R, for which:

$f|_{S(i)} = c_0+c\cdot j$, $j = 0, L$.

Without limiting the generality we could presuppose that the value of c is positive. We consider now the function $f : B(L)\rightarrow R$,, g from f^, for which we are given the values:

$g(00…0)=c_0 – 2$; $g|_{S(1)}=c_0+c-2$; $g|_{S(2)}=c_0+2c+1$;

The other values of g, so on the other elements of the partition, can be chosen arbitrarily so that we maintain the condition g from f^.

We presuppose that g is modular. Then, from the previous preposition we deduce that there exist a, b from R such that $g(x)=a+b\|x\|_1$. From $g(00…0) = c_0 – 2$, we get $a = c_0 – 2$. Also, from $g|_{S(1)} = c_0+c-2$ we get $b = c_0 + c – 3$. So, $g(x)= c_0 – 2+ (c_0 + c – 3)\cdot\|x\|_1$.

On the other hand, $g|_{S(2)}=c_0+2c+1$ so we find $c_0=9/2$. The presupposition that g would be unimodal is false and the proposition is thus proved.

Conclusion 3.1: Thus we proved that a modular function $f : B(L) \rightarrow R$, having the form $f(x) = c_0+c\|x\|_1$, c_0 and c belong to R, also covers among the elements of the class it generates unimodal functions but no modular ones. We know about the latter now that an average working time of (1+1)-EA, of the type $O(L\cdot logL)$ will correspond to them.

Now let $f : B(L) \rightarrow R$, have the form $f(x) = c_0+c\|x\|_1$, c_0 and c belong to R. and $g : B(L)\rightarrow R$, g from f^. From $f|_{S(i)} =c_0+c\cdot j$, j=0, L, we deduce

that obviously also g will have the same property, that is

$g|_{S(j)} = ct$, j=1, L-1.

We study now the next possibility: is there g unimodal but no modular, so that in the class generated by this there is no modular function? To answer this question, we will first prove a helping result.

Proposition 3.3. Let $f : B(L)\rightarrow R$, be unimodal, with $f|_{S(j)} = \alpha_j$, for any j=1, L-1. Then, for any i<j, we prove that $\alpha_i < \alpha_j$ or $\alpha_j < \alpha_i$, with i, j belong to {0, 1, …, L}.

Proof:

We absurdly presuppose that there exists I from {1, 1, …, L-1}such that $\alpha_i < \alpha_{i+1}$ and $\alpha_{i-1} < \alpha_i$ and let x belong to S(i). Then, from $f|_{S(i)} = \alpha_i$, for any i=1, L-1 and as D1(x) included in S(i+1)US(i-1), we get that for any y in D1(x), f(x)<f(y) so, so x is a local solution of f. As the choice of x in S(i) was arbitrarily chosen, we deduce that this is a local solution for f.

On the other hand, for any x and y in S(i), $\|x-y\|_1=2$, so the function f has more local solutions. This contradicts the unimodality condition for f and the proposition is thus proved.

Now, let $f : B(L) \rightarrow R$, unimodal, with $f|_{S(j)} = \alpha_j$, for any j=1, L-1, for which $\alpha_0< \alpha_1< … < \alpha_L$. Let $g : B(L) \rightarrow R$, $g(x)= c_0+c\|x\|_1$. We prove that there is a choice of the coefficients c_0 and c so that g belongs to the class f^.

$g\in$ f^ $\Rightarrow$ $g|_{S(0)}< g|_{S(0)} < g|_{S(1)}< … < g|_{S(L)}$, that is: $c_0< c_0+c<c_0+2c< … < c_0+Lc$. This is possible indeed, for c>0.

The condition $\alpha_0< \alpha_1< … < \alpha_L$ is not restrictive, in the sense that if the contrary inequality takes place, we choose c<0.

Thus we showed the next result, which gives answers for the questions we previously asked:

Proposition 3.4. The classes generated by the modular functions having the form $f : B(L) \rightarrow R$, with $f(x)=c_0+c\|x\|_1$, c_0 and c belong to R, cover all the unimodal functions $g : B(L) \rightarrow R$ having the property $g|_{S(j)}=ct$, j= 1, L-1.

Because the class measured by a function f like the one in the enounce is completely described by the inequality between the values of the function, it is obvious that $f(x)=c_0+c\|x\|_1$ can generate only two classes, for the distinct cases c<0 and c>0.

Observation 3.1. The modular functions we have discussed so far, as well as the unimodal elements of the classes they generated, have the particularity that they reach their minimum either in 00…0 (for c>0), or in 11…1 (for c<0).

So, a natural question arises: how do the previous conclusions structure themselves for a modular function that reaches its minimum in a point x* from B(L) arbitrarily chosen, so not especially 00…0 or 11…1? For such a function could we find in certain cases another partition of the space of the states in relation to which the passing matrix P might be comassable? We shall prove that the answer for this question is positive.

The first step towards this generalization is to watch the already obtained results, from another angle. The modular function $f(x)=c_0+c\|x\|_1$ reaches its minimum in 00…0 for c>0 and in its binary complement, for c<0. The partition in relation to which the passing matrix P will be comassable, is S(0), S(1), …, S(L), that is D(00…0). The natural generalization that results from this point of view is the following:

Let $f : B(L) \to R$ be unimodal, with x* from B(L) local/global solution. If the real constants $\alpha_0, \alpha_1, \ldots, \alpha_{L-1}$ exist so that

$f\big|_{Di(x^*)} = \alpha_i,\ i = 1, L-1$ (3.1.)

and if $i < j$ we have $\alpha_i < \alpha_j$, or $\alpha_j < \alpha_i$, then the passing matrix P associated with the Markov chain of (1+1)-EA and with the function f is comassable in relation to the partition D(x*).

We consider the modular function $f : B(L) \to R$, having the property that x* from B(L) is a global minimum and $f\big|_{Di(x^*)} = \alpha_i,\ i = 1, L-1$, $f(x) = c_0+c_1x_1 +c_2x_2 \ldots +c_Lx_L$.

From x* belongs to B(L), we get that there exists j from {0, 1, …, L} so that x* can be written as $x^* = \sum e_k$, where $k \in M$ a subset of {1, …, L}, with CardM = j. Then, the minimum value of f is:

$f(x)= c_0+\sum c_k = f^*,\ k \in M$

From the working hypothesis we have: $f\big|_{D1(x^*)} =$ ct., where:

$D_1(x^*) = \{x \in B(L)\ \big|\ x = x^*+ e_k, k \in M$ arbitrary chosen$\}$ U $\{x \in B(L)\ \big|\ x = x^*+ e_k, k \notin M$ arbitrary chosen$\}$

Let $i_1, i_2 \in M, i_1 \neq i_2$, be arbitrarily chosen and let: x = x*+e_{i1} and y = x*+e_{i2}. Obviously, x, y $\in$ D1(x*) and from $f\big|_{D1(x^*)} =$ ct we get f(x) = f(y) and finally $c_{i1=} c_{i2}$, for any $i_1 \neq i_2$, with i_1, i_2 from M.

Let c be the common value of the coefficients c_i, $i \in M$. Then:

$f(x)= c_0+c\sum x_k + \sum c_j\, x_j$ where $k \in M$ and $j \notin M$.

Let $i_1, i_2 \notin M, i_1 \neq i_2$, be arbitrarily chosen and let: x = x*+e_{i1} and y = x*+e_{i2}.. Analogously, x, y $\in$ D1(x*) and we get $c_{i1=} c_{i2}$, for any $i_1 \neq i_2$, with $i_1, i_2 \notin M$..

Let c' be the common value of all the coefficients c_i, $i \notin M$.. Then:

$f(x)= c_0+c\sum x_k + c'\sum x_j$ where $k \in M$ and $j \notin M$.

But, because Card M = j, we get that Card(non)M = L-j and then $f(x^*) = c_0+cj = f^*$.

We can easily prove that c' = -c and this shows that we proved the following result:

Proposition 3.5. Let the modular function $f : B(L) \to R$, having the property that $x^* \in B(L)$ is a global solution for f. If $x^* = \sum e_k$, where $k \in M$ a subset of {1, …, L}, with CardM = j and if $f\big|_{Di(x^*)} = \alpha_i, i = 1, L-1$, then:

$f(x)= c_0+c(\sum x_k - \sum x_j)$ where $k \in M$ and $j \notin M$.

Similar arguments but with a more complicated writing make all the previously proved results, for unimodal functions with the local solution 00…0, remain valid for the general case as well. Each modular function having the form of the function from the previous proposition generates two distinct classes of unimodal functions: for c>0 we obtain the class of functions that have the global solution x*, and for c<0 the result is the class of unimodals with the global solution x**. Moreover, any unimodal function g having the property $g\big|_{Di(x^*)} = \alpha_i, i = 1, L-1$, will belong to one of these two classes.

Because all the functions $g \in \hat{f}$ have the same global solution as that of f, we exclude the overlaying of two classes generated by modular functions having the property (3.1.) and distinct global solutions.

A new conclusion that results from this paper is:

Conclusion 3.2. Let $f : B(L) \to R$ be a unimodal function, with x* local/ global solution. If $f\big|_{Di(x^*)} = \alpha_i, i = 1, L-1$ and if $\alpha_i < \alpha_j$ for any I, j from {0, 1, …, L}, then the passing matrix of the Markov chain for (1+1)-EA can be comassed in relation to the partition D(x*) of the space of the states.

Proof:

Unlike the case that deals with the mutation matrix, whose elements do not depend on the objective

function, the study of the comassability of P will be performed after making various choices for f.

We will present the proof for this result for the particular case when $x^0 = 00\ldots0$. This will not restrain the generality of the result, but it will make writing easier, the proof for the general case following the same steps in fact. So, we will show that: If $f\big|_{S(i)} = \alpha_i$, i= 1, L-1 and if for any I, j from $\{0, 1, \ldots, L\}$, i≠j we have $\alpha_i < \alpha_j$, then the passing matrix of the Markov chain for (1+1)-EA can be comassed in relation to the partition $D(00\ldots)$ of the space of the states.

Let $x \in B(L)$, $x \in S(i)$, be for an index i arbitrarily fixed, i = 1, L. Then, we notice that for $y \in B(L)$, we have: $f(y) < f(x)$ iff $y \in S(j)$ with j>i.

Let i, j $\in \{1, 2, \ldots, L\}$ be two arbitrary indices and let S(i), S(j) be the sets of the partition. Let $x \in S(i)$. We show that p(x, S(i)) does not depend on particularly choosing x belonging to S(i). So, we compute p(x, S(j)), where p(x, S(j)) = $\sum$p(x, y) where $y \in S(j)$. There are to be discussed three cases: j=i, if i < j and i > j. In each situation, we compute p(x, S(i)) = $\sum$p(x, y) where $y \in S(i)$ and we find that p(x, S(i)) acquires a value independent from the choice we make for x. We then prove that the matrix P is comassable in relation to D(00...0), q.e.d.

Observation 3.2. The condition $\alpha_i < \alpha_j$, for i < j, imposed in this enounce, is not essential the comassability property, but for describing the unimodality.

Conclusion 3.3. Any unimodal function f : B(L) $\rightarrow$ R, with x* local/ global solution, with $f\big|_{Di(x*)} = \alpha_i$, i= 1, L-1 and for which any i<j chosen from $\{0, 1, \ldots, L\}$ lead to $\alpha_i < \alpha_j$, is part of a class generated by a modular function having the same properties.

4. Conclusions

In this paper we focused our study on the issue of comassability of states for a matrix describing an Evolutionary Algorithm. We found that a modular function f : B(L) $\rightarrow$ R, having the form $f(x) = c_0 + c\|x\|_1$, c_0 and c belong to R, covers among the elements of the class it generates unimodal functions but no modular ones.

We also found that if f : B(L) $\rightarrow$ R is an unimodal function, with x* local/ global solution. If $f\big|_{Di(x*)} = \alpha_i$, i= 1, L-1 and if any i<j chosen from $\{0, 1, \ldots, L\}$ lead to $\alpha_i < \alpha_j$, then the passing matrix of the Markov chain for (1+1)-EA can be comassed in relation to the partition D(x*) of the space of the states.

Finally, we proved that any unimodal function f : B(L) $\rightarrow$R, with x* local/ global solution, with $f\big|_{Di(x*)} = \alpha_i$, i= 1, L-1 and for which any i<j chosen from $\{0, 1, \ldots, L\}$ lead to $\alpha_i < \alpha_j$, is part of a class generated by a modular function having the same properties.

We then were able to described some classes of fitness functions for which the (1+1) EA leads its optimum in a polynomial time, which is helpful both for direct using this optimization algorithm and for developing the study for more complicated situations.

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