



# Random Walk Theory and the Romanian Capital Market: A New Perspective

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## Abstract

Forecasting rates of return, thus the attempt to predict the behavior of financial assets, with an increased degree of accuracy, represents one of the most outstanding challenges for the academic and investment area. The main purpose of the paper is to analyze past fluctuations of the prices of security titles, taking into account the original hypothesis that they are influenced by past values of those prices, and of course, taking into consideration the fact that the amount of data an investor may possess is much richer than the amount of historical data, with respect to the rates of return time series.

In the end, some conclusions regarding the application of the random walk theory and the Romanian capital market efficiency were drawn, based on the results obtained from the statistical tests, and also, due to the fact that the market efficiency has, as a theoretical approach and mathematical model, the random walk theory.

**Keywords:** random walk, market efficiency, information, capital markets.

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## Introduction

In 1827 an English botanist, Robert Brown, noticed that small particles suspended in fluids perform peculiarly erratic movements. This phenomenon, which could also be attributed to gases, is referred to as Brownian motion. After that moment further, the theory has been considerably generalized and extended by Fokker, Planck, Burger, Ornstein, Uhlenbeck, Chandrasekhar, Kramers and others. On the purely mathematical side, various aspects of the theory were analysed by Wiener, Kolmogoroff, Feller, Levy, Doob (1939) and Fortet (1943). Even Albert Einstein had an important contribution to this theory.

The limitations of this theory were already recognized by Einstein and Smoluchowski (1916), but are often disregarded by other writers. An improved theory, known as „exact”, was advanced by Uhlenbeck and Ornstein (the Ornstein-Uhlenbeck Process) (1930) and by Kramers (1946). The random walk theory was first brought to light by the

discrete approach of Einstein-Smoluchowski, and it consists in treating Brownian motion as a discrete random walk. The main advantages of this discrete approach are pedagogical, but it may suggest various generalizations which will contribute to the development of the Calculus of Probability.

The random walk theory has nowadays a practical implication into the financial theory, stating that the stock prices evolve accordingly to a random walk, and thus they are impossible to predict. This theory is consistent with the efficient-market hypothesis. In finance, this theory is mainly linked by the name of Eugene Fama (1965), even if Burton Malkiel (1973) is considered to have strongly developed it.

## Methodology

The study was conducted by starting with reviewing the literature regarding the Brownian motion, Wiener process, Ito process, Ornstein-Uhlenbeck process and

reaching the random walk theory. Starting from the major theories, the random walk theory is presented under its major sub-hypothesis, starting from independent and identically distributed increments and reaching to dependent increments, but uncorrelated. In order to apply those hypothesis, there were used time series extracted from the daily closing prices for the Company of Financial Investment Services (SIF5), for the period starting from the 5th of January, 2009, and ending to the 14th of February, 2012. All the three sub-hypothesis of the random walk theory are tested using different statistical tests, in order to determine the application and the compliance of this theory with the Romanian Stock Market.

### **Random Walk Theory – Major Sub-Hypothesis**

Basically, a market is defined to be information – efficient if no investors can reach abnormal systematic earnings and, also, the true expected return of any security is equal to its equilibrium expected value (Fama, 1976). From the first point of view, the main concern for the market is to give equal chances to each investor, which means that there are no investors able to gain every time and investors to lose every time. From the second opinion, it is important for markets to work, thing that will have as a result a right estimation of asset returns. In this context, there were many trials to develop instruments for testing market information efficiency. Many investigation techniques used in order to test the possibility of earning abnormal returns were revealed. In this sense, Kendall (1956) and Alexander (1961) turned to tests of the serial correlation; Fama and Blume (1966) appealed to simple trading rules tests; Jagadeesh (1990) and Jagadeesh and Titman (1993) resorted to overreaction tests; DeBondt and Thaler (1985), Poterba and Summers (1998) and Fama and French (1988) fell back upon tests of long-horizon return predictability.

Campbell, Lo and MacKinley (1997) stated that „any test of efficiency must assume an

equilibrium model that defines normal security returns. If efficient hypothesis is rejected, this can be because the market is truly inefficient or because an incorrect equilibrium model has been assumed”.

Fama (1970) stated that a market is information efficient if prices fully reflect all the available information from the market.

The notion of market efficiency can be as follows: the more efficient the market is, the more aleatory the sequence of price changes generated by the market is (Dragota, Stoian, Pele, Mitrica, Bensafta, 2009).

At least with the emerging markets, such as East European Ex-communist Countries, due to some of their particular features, such as lack of liquidity, econometric tests could be distorted (Pele and Voineagu, 2008). The informational efficiency of the Romanian capital market was differently tested in the past years. From this point of view, most of the studies were related to the possibility of gaining abnormal earnings (Dragota, Caruntu, Stoian, 2006).

Similar studies were done for other ex-communist countries. For instance, Chun (2000) based on variance ratio tests found that the Hungarian capital market was weakly efficient; Gilmore and McManus (2003) investigated informational efficiency in its weak form from the Czech Republic, Poland and Hungary (within 1995-2000) and rejected the random walk hypothesis based on the results of a model comparison approach.

Consequently, the statistical manner to express the market efficiency is the random walk hypothesis (RWH), which can be formulated in three different sub-hypothesis, respectively: independently and identically distributed increments, independent increments, and uncorrelated increments. Those sub-hypothesis start from a less broad perspective, getting to a more relaxed and natural perspective. Those hypothesis are further presented:

**RW1 Hypothesis: Independent Increments, Identically Distributed**

The most natural way of expressing the random walk hypothesis is the one in which the price of financial assets is represented by a stochastic process, following an internal dependency of the manner:

$$P_t = \mu + P_{t-1} + \varepsilon_t \quad (1)$$

Where  $\varepsilon_t \sim WN(0, \sigma^2)$  represents a white noise, a series of independent random variables, identically distributed:

$$\begin{aligned} E[\varepsilon_t] &= 0, \forall t \\ \text{Var}[\varepsilon_t] &= \sigma^2, \forall t \\ \varepsilon_t \text{ si } \varepsilon_{t+k} \text{ sunt variabile independente, } \forall k &\neq 0 \end{aligned}$$

More,  $\text{cov}[\varepsilon_t, \varepsilon_{t+k}] = 0$  and  $\text{cov}[\varepsilon_t^2, \varepsilon_{t+k}^2] = 0, \forall k \neq 0$ .

In equation (1),  $P_t, P_{t-1}$  represent the price values for two successive time moments, and  $\mu$  represents the expected price movement, the so-called *drift*.

The most common condition for the random variable  $\varepsilon_t$  is the fact it follows a normal distribution function, except the fact that it represents a white noise, condition that generates a certain formal commonness. But this may be the cause of appearing certain irregularities with the practice, due to the fact that the normal distribution function covers the whole range of real numbers, and it may result into the fact that there may be a non-zero probability that the price of a security title to be negative. A way of avoiding this fact may be by using instead of stock prices series, the series of natural logarithms of those prices:  $p_t = \log P_t$ .

Model RW becomes then a log-normal model:

$$p_t = \mu + p_{t-1} + \varepsilon_t \quad (2)$$

Where  $\varepsilon_t \sim WN(0, \sigma^2)$  represents a white noise.

**RW2 Hypothesis: Independent Increments**

Even the simplicity and the elegance of the RW1 Model seem very alluring, the supposition of the existence of identically distributed independent increments is not quite natural.

The influencing factors that determine the evolution of the prices of financial assets are not always the same and do not affect those prices with the same intensity. Also, the economic conditions vary much during time, this making the hypothesis of the existence of the same distribution function over time to be not natural. (1)

Then, the RW2 model derives directly from the RW1 model, but the single difference resides in ignoring the hypothesis of the same distribution function of the random variable  $\varepsilon_t$ :

$P_t = \mu + P_{t-1} + \varepsilon_t$ , where  $\varepsilon_t$  is a series of random variables:

$$\begin{aligned} E[\varepsilon_t] &= 0, \forall t \\ \text{Var}[\varepsilon_t] &= \sigma_t^2, \forall t \\ \text{cov}[\varepsilon_t, \varepsilon_{t+k}] &= 0 \quad \text{and} \quad \text{cov}[\varepsilon_t^2, \varepsilon_{t+k}^2] = 0, \forall k \neq 0. \end{aligned}$$

Even RW2 model is weaker than the RW1 model, the former keeps the essence of the latter: every future movement of the stock prices is unpredictable, using the past price movements.

**RW3 Hypothesis: Uncorrelated Increments**

Relaxing the hypothesis of the above-described models, we can obtain a more generalized form of the random walk hypothesis, in which increments are dependent, but uncorrelated.

$P_t = \mu + P_{t-1} + \varepsilon_t$ , where  $\varepsilon_t$  is a series of random variables: (2)

$$\begin{aligned} E[\varepsilon_t] &= 0, \forall t \\ \text{Var}[\varepsilon_t] &= \sigma_t^2, \forall t \\ \text{cov}[\varepsilon_t, \varepsilon_{t+k}] &= 0 \end{aligned}$$

In order to test the RW1 hypothesis, it was used the Runs Test, while for testing the RW3 hypothesis, one of the most natural ways was to detect some possible serial correlations, correlations existent between the values of a time series in different moments in time.

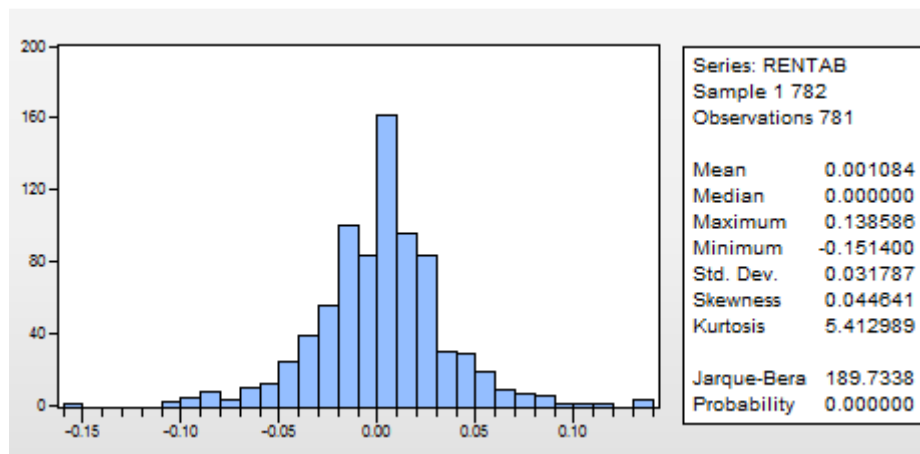
Following the conditions of the weakest random walk hypothesis, RW3, first order differences are uncorrelated for every time interval; as a consequence, for testing the RW3 hypothesis, we will observe the values the autocorrelation coefficient takes in different time moments. A powerful test, that may detect the evidence of the RW1 hypothesis, is produced by the Q-Statistics, introduced by Box and Pierce (1970). Ljung and Box (1978) have offered an alternative to this test, for small-size samples.

An important property of all random walk hypothesis is the fact that the variance of the residuals must be a function varying linearly with respect to time. As a

consequence of this fact, the random walk hypothesis may be tested with the variances' ratio (Multiple Variance ratio Test). In order to make a decision regarding the acceptance or the rejection of the random walk hypothesis, the Multiple Variance Ratio (MVR) approach was used (Chow and Denning, 1993).

In order to test different aspects regarding the behavior of financial assets, we have used daily closing prices for the Company of Financial Investment Services (SIF5), for the period starting from the 5<sup>th</sup> of January, 2009, and ending to the 14<sup>th</sup> of February, 2012. Based on those data, we have computed the daily rates of return, using the every day closing prices, by the formula:  $r_t = \ln \frac{P_t}{P_{t-1}}$ , where  $P_t$  represents the daily closing price of day  $t$ .

### Descriptive Statistics and Verifying the Normal Distribution of the Daily Rates of Return



Analyzing the indicators of the daily returns distribution, we can draw the following conclusions:

- In all the cases, the Gaussian distribution hypothesis cannot be accepted, due to both the values of the kurtosis coefficient,

and to the values of the Jarque-Berra Statistics.

- The distribution of the returns is leptokurtic, different form the shape of a standard normal distribution.

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.083	0.083	5.3390	0.021
		2	0.033	0.027	6.2106	0.045
		3	0.021	0.017	6.5661	0.087
		4	0.016	0.012	6.7554	0.149
		5	0.066	0.064	10.219	0.069
		6	0.051	0.040	12.299	0.056
		7	-0.017	-0.029	12.529	0.084
		8	0.039	0.038	13.746	0.089
		9	0.059	0.052	16.481	0.057
		10	0.002	-0.013	16.484	0.087
		11	0.034	0.026	17.404	0.096
		12	0.044	0.040	18.957	0.090
		13	-0.026	-0.038	19.481	0.109
		14	0.043	0.034	20.930	0.103
		15	0.116	0.111	31.742	0.007
		16	0.001	-0.021	31.743	0.011
		17	0.044	0.028	33.313	0.010
		18	-0.042	-0.050	34.707	0.010
		19	0.012	0.016	34.828	0.015
		20	-0.077	-0.106	39.624	0.006
		21	-0.023	-0.017	40.052	0.007
		22	0.033	0.047	40.942	0.008

In order to verify the random walk hypothesis for the daily rates of return for the Company SIF 5, we have applied the

Runs Test in SPSS and the Multiple Variance Ratio Test in Eviews, in both cases with or without homoskedasticity.

**Table 1. Random Walk Hypothesis for Daily Rates of Return of the Company SIF5**

Descriptive Statistics		rentab	Runs Test 2		rentab
N		781	Test Value <sup>a</sup>		.00108400
Mean		.00108400	Cases < Test Value		414
Std. Deviation		.031786591	Cases >= Test Value		367
Minimum		-.151400	Total Cases		781
Maximum		.138586	Number of Runs		379
Percentiles	25th	-.01604324	Z		-.797
	50th (Median)	.00000000	Asymp. Sig. (2-tailed)		.426
	75th	.01886164			
			a. Mean		

Runs Test		rentab	Runs Test 3		rentab
Test Value <sup>a</sup>		.000000	Test Value <sup>a</sup>		.000000
Cases < Test Value		339	Cases < Test Value		339
Cases >= Test Value		442	Cases >= Test Value		442
Total Cases		781	Total Cases		781
Number of Runs		383	Number of Runs		383
Z		-.124	Z		-.124
Asymp. Sig. (2-tailed)		.901	Asymp. Sig. (2-tailed)		.901
			a. Mode		

The value Asymp.Sig(2-tailed) (superior to the level of 5%) corresponding to the z test for the cutting point does not put into evidence the rejection of the null hypothesis, according to which the daily

rates of return follow a random walk process, that may conduct to the conclusion that, indeed, these daily rates of return follow a random walk movement, according to the results obtained from the Runns test.

**Table 2. Multiple Variance Ratio Test for the RW1 hypothesis  
(With the Supposition that the Homoskedasticity Condition is being Fulfilled)**

Null Hypothesis: Log CLOSE is a random walk  
Date: 04/24/11 Time: 16:56  
Sample: 1 782  
Included observations: 781 (after adjustments)  
Standard error estimates assume no heteroskedasticity  
Use biased variance estimates  
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max  z  (at period 16)*	3.324841	781	0.0035
Wald (Chi-Square)	12.32677	4	0.0151

Individual Tests				
Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	1.079993	0.035783	2.235527	0.0254
4	1.164664	0.066944	2.459741	0.0139
8	1.300674	0.105847	2.840649	0.0045
16	1.523680	0.157505	3.324841	0.0009

\*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = 0.00108399584296)

Period	Variance	Var. Ratio	Obs.
1	0.00101	--	781
2	0.00109	1.07999	780
4	0.00118	1.16466	778
8	0.00131	1.30067	774
16	0.00154	1.52368	766

The Chow-Denning Statistics of 3.32 is associated to the period 16 of the individual tests. P-value probability of 0.0035 conducts to rejecting the null hypothesis of random walk. The results are similar also

for the Wald Test for the common hypothesis. The individual statistics conduct to the rejection of the null hypothesis, the p-value probability being inferior to the level of 0.05.

**Table 3. Multiple Variance Ratio Test for the RW3 Hypothesis  
(Supposing that the Heteroskedasticity Condition is being Fulfilled)**

Null Hypothesis: Log CLOSE is a martingale

Date: 04/24/11 Time: 17:12

Sample: 1 782

Included observations: 781 (after adjustments)

Heteroskedasticity robust standard error estimates

User-specified lags: 2 4 8 16

Test probabilities computed using wild bootstrap: dist=twopoint,  
reps=5000, rng=kn, seed=1000

Joint Tests	Value	df	Probability
Max  z  (at period 16)	2.720065	781	0.0226

Individual Tests				
Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	1.082766	0.052288	1.582899	0.1090
4	1.173669	0.094558	1.836632	0.0700
8	1.324322	0.146109	2.219727	0.0286
16	1.583978	0.214693	2.720065	0.0138

Test Details (Mean = 0.00108399584296)

Period	Variance	Var. Ratio	Obs.
1	0.00101	--	781
2	0.00109	1.08277	780
4	0.00119	1.17367	778
8	0.00134	1.32432	774
16	0.00160	1.58398	766

Probabilities p-value of the individual tests, which have been generated using the wild bootstrap technique are in general consistent with the previous results, even if they are relatively higher than before. The individual test for the period 2, which was significant in the case of homoskedasticity, becomes insignificant for a significance level of 5%. The result of the Chow-Denning Test is 2.72 with a p-value probability of 0.02, which may conduct to the rejection of the null hypothesis that log close is a martingale.

### Conclusions

After performing and running the statistical tests, some major conclusions may be drawn:

- The Runs Test, even if it has a limitative explanatory power, conducts to the

general acceptance of the random walk hypothesis of the daily closing price of the stock share of company SIF5;

- The Multiple Variance Ratio Test conducts to the rejection of the RW1 hypothesis, supposing that the homoskedasticity condition is being fulfilled;
- The Multiple Variance Ration Test conducts to the rejection of the RW3 hypothesis, supposing that heteroskedasticity condition is being fulfilled.

Based on the results discussed above, it is difficult to state if the Romanian capital market is informational efficient in its weak form.

These results sustain the hypothesis that the Romanian capital market improved its

performance over the last few years. Also, the Romanian investors' professional experience increased, and probably, their ability to evaluate assets in an appropriate manner has developed. All these conclusions revealed from the study related the fair game on the Romanian capital market, which is in accordance with Pele and Voineagu (2008) conclusions.

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